

Faraday's Law of Electromagnetic Induction

By Aaron Logan

Faraday's Law: an Overview and Brief History

Faraday's Law of Electromagnetic Induction is one of several equations that govern electromagnetic phenomena. These equations for electromagnetics are extremely important since they provide a means to precisely describe how many natural physical phenomena in our universe arise and behave. The ability to quantitatively describe physical phenomena not only allows us to gain a better understanding of our universe, but it also makes possible a host of technological innovations that define modern society. Understanding Faraday's Law of Electromagnetic Induction can be beneficial since so many aspects of our daily life function because of the principles behind Faraday's Law. From natural phenomena such as the light we receive from the sun, to technologies that improve our quality of life such as electric power generation, Faraday's Law has a great impact on many aspects of our lives.

Faraday's Law is the result of the experiments of the English chemist and physicist Michael Faraday (figure 1). The concept of electromagnetic induction was actually discovered simultaneously in 1831 by Faraday in London and Joseph Henry, an American scientist working in New York (Sadiku, 386), but Faraday is credited for the law since he published his work first (Wikipedia). An important aspect of the equation that quantifies Faraday's Law comes from the work of Heinrich Lenz, a Russian physicist who made his

contribution to Faraday's Law, now known as Lenz's Law, in 1834 (Institute of Chemistry).

Faraday's law describes electromagnetic induction, whereby an electric field is induced, or generated, by a changing magnetic field. Before expanding upon this description, it is necessary to develop an understanding of the concept of fields, as well as the related concept of potentials.

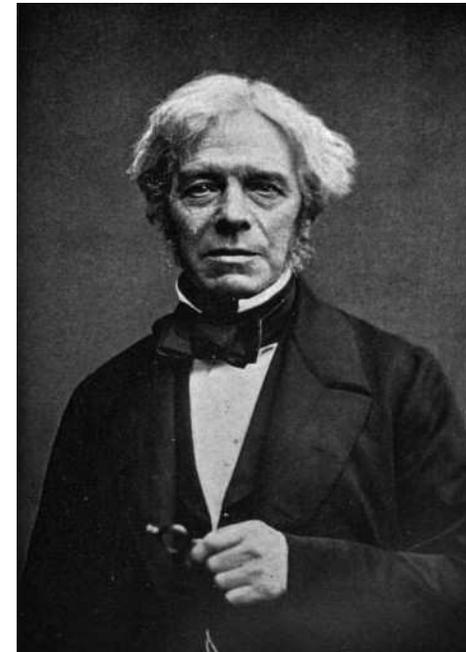


Figure 1: Michael Faraday (1791 – 1867) is credited for discovering the principle of electromagnetic induction. *Photo: Wikipedia*

Fields and Potentials: Essential Electromagnetic Concepts

Fields are a fundamental concept in physics, and are especially important for describing electromagnetic phenomena. But what are they? A field is an abstract concept used to describe how particular forces act upon certain particles or bodies in a region. These forces are often, but not necessarily, the result of some other body or particle present in that region. For example, the earth is a source of a gravitational field, and all objects “near” to the earth, including yourself, aircraft in the sky, satellites, and even the moon, sun, and distant planets experience a gravitational force of attraction toward the earth. “Near” means that the distance between the two objects is small enough for the gravitational force of attraction between them to be non-trivial. Similarly, electric fields, existent around electric charges, exert forces on any other electric charges in their presence. Charges that are far enough away from the field source may be regarded to be, as physicists say, “at infinity,” such that the force acting on them by the field is negligible.

Fields are described as vectors. A vector is a way of quantifying something with both magnitude and direction. For example, wind is best described as a vector: “today the wind averaged ten miles per hour due east.” In the case of electric fields, magnitude at a certain point is given as the force exerted per unit charge, in newtons per coulomb (N/C), or, equivalently, volts per meter (V/m). The field vector direction is, by convention, the direction a positive charge would be accelerated by the field at that point. This means that negative charges, with charge polarity opposite that of positive charges, are accelerated against the direction of an electric field.

Lightning, a familiar natural phenomenon, provides a good illustration of the electric field, its vector representation, and the forces exerted on charges by the field (figure 2). Prior to a lightning strike, an electric field exists between the earth and the thunderhead, directed toward the sky due to a buildup of negative electrons on the bottom of the thunderhead. Near the earth the electric field strength can be as high as 100N/C (Sadiku, 305). This field exerts a downward force on the electrons, against the direction of the field, and is strong enough to pull them down to earth in what we call a lightning bolt.

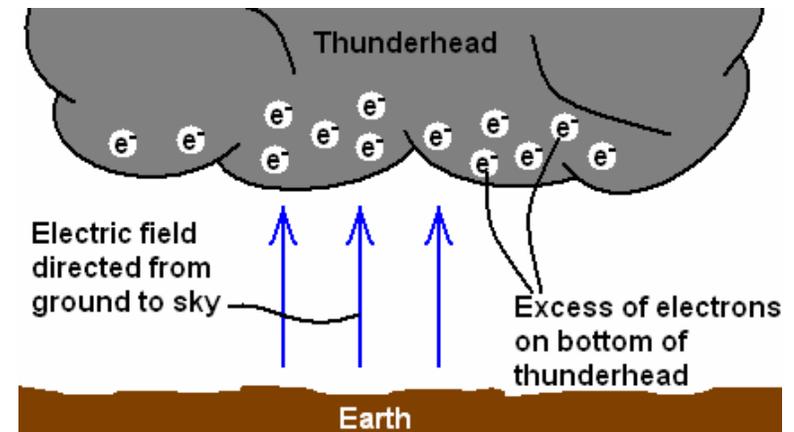


Figure 2: An electric field exists between the earth and a negatively charged thunderhead and is directed toward the sky. The use of arrows to denote the electric field direction reminds us that this field is a vector. The electrons are acted upon in a direction opposite the field direction (i.e. downward) since they are negatively charged.

Potentials, associated with all fields, are another important physical concept essential to electromagnetics. A potential exists between any two points in a field such that work must be done to move an object from one point to the

other. Returning to the gravity example, there is no potential energy difference between two people standing fifty feet apart on a flat driveway: the gravitational field does no work on either of them as they walk toward one another. There is, however, a potential energy difference between a skier standing at the bottom of a hill and another standing at the top. The skier at the top had to do work *against* the gravitational field to get there, but in turn, the gravitational field does work on the skier in pulling him or her to the bottom. Likewise, when two charges are separated in an electric field such that a gradient or “slope” separates them, they are separated by an electric potential.

You have likely already been exposed to the concept of electric potential, as it is more commonly referred to as “voltage.” The common AA cell, rated 1.5 volts, has an electric field directed from the positive electrode to the negative electrode (figure 3). Since the electrodes are at different points in the electric field, there is a potential, or voltage, between them. This potential means the cell has the capacity to do work on electric charges. When an electric load, such as a light bulb or a CD player, is connected to the cell’s terminals, the cell’s electric field forces charges (electrons) through the load, causing them to transfer energy to the load as they return to the “downhill” side of the cell.

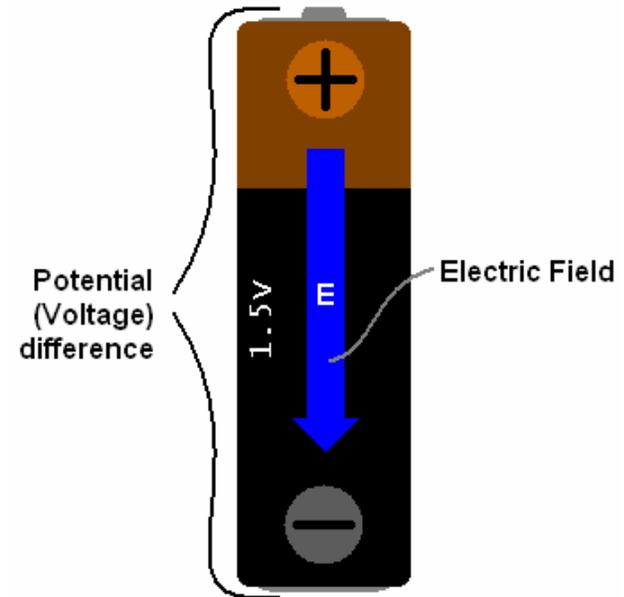


Figure 3: in a AA cell, the positive and negative electrodes are separated by a field which creates a potential difference, or voltage, between them. This voltage represents the electric field’s capacity to force charges through an external circuit.

Faraday's Law: an Explanation

Having covered the basics of electric fields and potentials, we can explore Faraday's Law in order to gain a qualitative understanding of the concepts it conveys.

Listed below are the two mathematical forms of Faraday's Law: the *point* or *differential form*, and the *integral form*. Although the two forms appear highly distinct, they mean exactly the same thing and can be used interchangeably in calculations. The point form equation can be transformed to the integral form equation and vice versa by the application of a single vector calculus theorem¹. Although synonymous, the two forms of the law lend themselves to different conceptual understanding depending on the physical context. Physicists and electrical engineers often like to state Faraday's Law in the more compact point form, but prefer using the integral form for calculations since it is more physically intuitive.

Point / Differential Form	Integral Form
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{S}$

Table 1: The two mathematical forms of Faraday's Law.

¹ *Stoke's Theorem* equates the curl of a vector field \vec{A} to a circulation line integral of the vector field by the application of a surface integral:

$$\oint \vec{A} \cdot d\vec{l} = \int (\nabla \times \vec{A}) \cdot d\vec{S}.$$

Avoiding the drudgery imposed by attempting to understand the sundry mathematical symbols, Faraday's law says that a time-varying magnetic field induces an electric field (Sadiku, 386). More formally, here is the essence of Faraday's Law:

The sum of all electric field components tangent to a closed spatial path, or "loop," is equal to the *negative* time-rate of change of the magnetic flux through the surface bounded by that path.

First, let's understand what is meant by "flux." Imagine water flowing through a pipe in which a screen spans the cross-section (figure 4). The flow of water *across* the screen can be considered "flux." Similarly, magnetic flux refers to a magnetic field intersecting a surface.

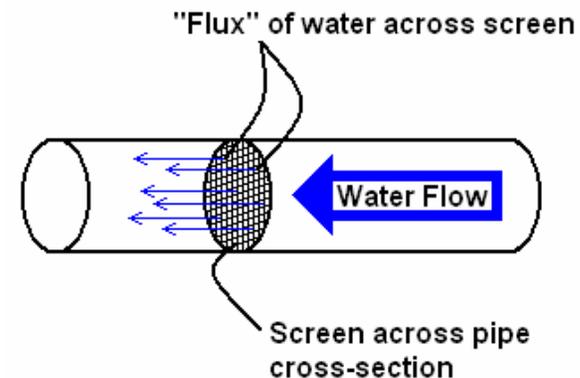


Figure 4: the concept of flux can be understood by considering water flowing through a screen.

Now imagine that, as the water flows through the screen in the pipe, its rate of flow increases, i.e. it accelerates. This means that the time-rate of change of the water flow is positive relative to the direction of flow. On the other hand, if the flow rate decreases, then this time-rate of change is negative with respect to the direction of flow. The same applies to the magnetic flux through a surface: if its magnitude is increasing with respect to the field's direction, then the time-rate of change is positive; otherwise, it is negative. The value of this rate of magnetic flux change is what is conveyed by the right sides of the equations in table 1.

Now direct your attention to figure 5. In this figure, there is a changing magnetic field, represented by the red arrow “coming out of” the page. Surrounding this changing magnetic field (and flux) is an arbitrary closed path along which are marked several tangential electric field components $E_1, E_2, \dots, E_i, \dots, E_{n-1}, E_n$. The sum of these components, relatively speaking, is what is indicated by the left sides of the equations in table 1.

Faraday's Law equates the two last concepts: the total electric field summed around a closed path (the left side of the equation) is equal to the time-rate of change of the magnetic flux

through the surface bounded by that path (the right side of the equation). Physically, this means a time-changing magnetic flux gives rise to an electric field in its neighborhood. Recall from our earlier discussion that there must be an electric potential, or voltage, associated with every electric field. Thus our understanding of Faraday's law can be extended to say that *a time-variant magnetic field induces an electric potential or voltage.*

Finally, what is the meaning of the negative sign in the Faraday's Law equation? This is the result of Lenz's Law, which relates the polarity of the induced electric field and

potential to the direction of the magnetic flux change. Lenz' Law states that an electric potential generated by a changing magnetic flux has a polarity such that it *opposes* the time-change of the magnetic flux from which it originated.

Why “opposes?” Well, this is essentially the *conservation of energy* in action. Let us perform a simple thought experiment: suppose for a moment that the induced electric potential did not oppose its source (a changing magnetic field). According to Ampere's Law, the counterpart of Faraday's law in which the roles of the magnetic and electric fields are transposed, the induced electric field will in turn induce a magnetic field.

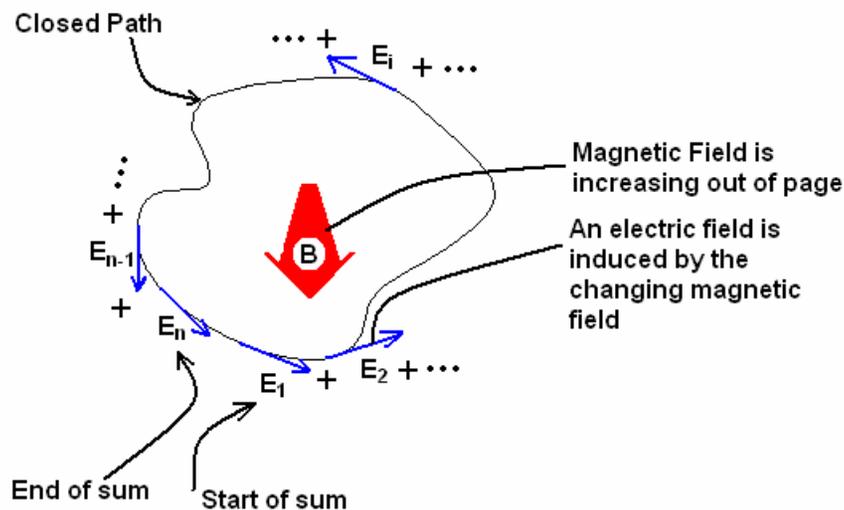


Figure 5: Summing the *tangential* electric field components around a closed path produces the left sides of the equations in table 1. In this case, the path is arbitrary; for applications of Faraday's Law, the path may be chosen along actual physical boundaries. The magnetic field, spread across the surface bounded by the path, constitutes a magnetic flux. In this picture, the magnetic field and flux are increasing out of the page, resulting in the counterclockwise electric field shown. This is in accordance with Lenz's Law, as discussed at left.

Superimposing this magnetic field on the original one would cause the two to add, rather than subtract, since they are not, as we are currently supposing, in opposition. The resulting increase in the magnetic field would further add to the induced electric field, setting off a cycle in which both fields feed each other's growth ad infinitum. This would result in infinite energy density in the electric and magnetic fields, which is a physically impossible situation as it violates the conservation of energy. Thus, our supposition that the induced electric field does not oppose its source was incorrect, validating Lenz's Law.

With this conceptual understanding of Faraday's Law, it is possible to appreciate its profound impact on our world.

Faraday's Law: Consequences

Faraday's Law is so fundamental to the workings of our universe that if the truths it conveys were not so, it is difficult to imagine how the universe as we know it would be different. One could say that electromagnetic waves wouldn't exist, and without these, perhaps, the universe wouldn't either. Or, perhaps, life would continue, but in a dramatically different way than what we experience. But this is a discussion best left for philosophers. What we do know for certain is that mankind's *understanding* of these laws has had a colossal impact on how we live in our world today: various inventions and technologies that incorporate Faraday's Law have revolutionized mankind's living for well more than a century.

As mentioned earlier, the counterpart of Faraday's Law is Ampere's Law. Together, these two laws describe the interaction of dynamic electric and magnetic fields. These

laws describe how *electromagnetic (EM) waves* are generated and, with the help of two other electromagnetics equations, propagated through various media. EM waves are essential to our existence and to our quality of life. EM waves of many different frequencies are responsible for myriads of different phenomena: low frequency EM waves are used for radio transmissions and television broadcasts; low- to mid-frequency microwaves are used in satellite and mobile communications and in microwave ovens; mid-frequency infrared radiation from the sun (figure 6) heats our planet; mid-frequency visible light waves allow us to see and makes plant and animal life on earth possible; mid-frequency ultraviolet radiation is enjoyed by tanning sunbathers; high-

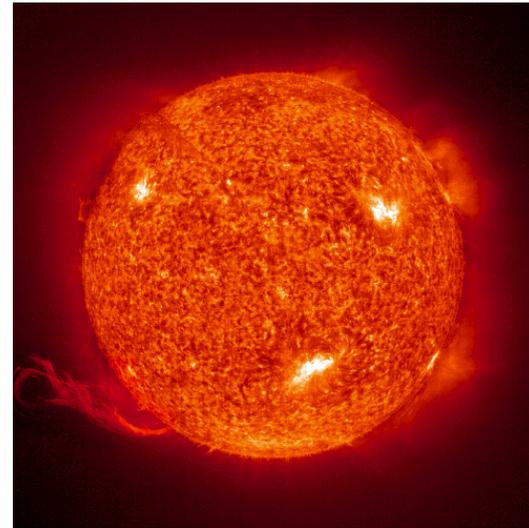


Figure 6: Faraday's Law is in part responsible for electromagnetic waves, which have many occurrences in nature and are indeed fundamental to how we live. The sun is a source of electromagnetic waves of many different frequencies, including infrared and visible light.

Photo: Harvard-Smithsonian Center for Astrophysics

frequency x-rays are used in medical diagnostic equipment and in materials analysis; and ultra-high frequency gamma radiation is involved in subatomic phenomena.

In addition to the many natural phenomena that stem from Faraday's Law, there are many technological applications of electromagnetic induction, some of which have already been listed. Another application of electromagnetic induction that has had a tremendous impact on the way the society functions is electric power generation. The electric generator (figure 7) uses electromagnetic induction by rotating windings (loops of wire) in a magnetic field. As the windings rotate through the field, a time-varying flux is incident across them, resulting in an induced voltage. The current flow that results from applying this voltage to the electric transmission grid allows for the distribution of electric power to customers far and wide.



Figure 7: the electric generator is a direct application of Faraday's Law of electromagnetic induction. Here are two synchronous generators on the floor of a generation plant.
Photo: Plant Operations Division @ the University of Michigan

Conclusion

Faraday's Law of Electromagnetic Induction, first observed and published by Michael Faraday in the mid-nineteenth century, describes a very important electromagnetics concept. Although its mathematical representations are cryptic, the essence of Faraday's is not hard to grasp: it relates an induced electric potential or voltage to a dynamic magnetic field. This concept has many far-reaching ramifications that touch our lives in many ways: from the shining of the sun, to the convenience of mobile communications, to electricity to power our homes. We can all appreciate the profound impact Faraday's Law has on us.

References

"Close Encounters of the Stellar Kind." Chandra Chronicles: Gateway to the Universe of X-Ray Astronomy. 20 Apr. 2007. Harvard-Smithsonian Center for Astrophysics. 17 Feb. 2008 <<http://chandra.harvard.edu/chronicle/0303/ce/>>.

"Electric Generator." Plant Operations Division @ the University of Michigan. Feb. 2001. Central Power Plant, University of Michigan. 18 Feb. 2008 <http://www.plantops.umich.edu/utilities/CentralPowerPlant/cogeneration/electric_generator.html>.

"Heinrich Friedrich Emil Lenz." Famous Scientists. 02 Apr. 2005. Institute of Chemistry: the Hebrew University of Jerusalem. 17 Feb. 2008 <<http://chem.ch.huji.ac.il/history/lenz.html>>.

"Michael Faraday." Wikipedia. 17 Feb. 2008 <http://en.wikipedia.org/wiki/Michael_Faraday>.

Sadiku, Matthew. Elements of Electromagnetics. 4th ed. New York: Oxford UP, 2007. 384-386.